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Black hole entropy and soft hair

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This paper summarizes the status of our long-term project on large diffeomorphisms, soft hair and the quantum structure of black holes until the end of our time together.

The Enigma of Black Hole Entropy

The Bekenstein-Hawking area-entropy law suggests that black holes possess a microscopic structure. However, the physical origin of this entropy for realistic, astrophysical Kerr black holes remains unknown. This work proposes an explanation rooted in spacetime symmetries.

$$S_{BH} = \frac{Area}{4}$$

(in Planck units)

A Clue from Hidden Conformal Symmetry

Generic Kerr black holes exhibit a hidden SL(2,R) conformal symmetry. This symmetry emerges not in a region of spacetime, but in the near-horizon region of ***phase space*** for low-energy waves.

$$\omega(r - r_+) \lesssim 1$$

The Thermodynamics of a Kerr Black Hole

The black hole’s thermodynamics, and its absorption cross-section for soft modes, can be described as two independent sectors—"left-moving" and "right-moving"—each with its own effective temperature. This motivates the hypothesis that the black hole can be described by a 2D CFT.

$$T_L = \frac{r_+ + r_-}{4\pi a} \qquad T_R = \frac{r_+ - r_-}{4\pi a}$$

Revealing the Symmetry with Conformal Coordinates

To make the hidden conformal symmetry explicit, a special coordinate system is introduced. These “conformal coordinates” regularize the geometry at the horizon’s bifurcation surface, where the past and future horizons meet.

$$w_+ = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T R \phi}$$

$$w_- = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T L \phi - t/2M}$$

Large Diffeomorphisms as Symmetry Generators

In these coordinates, an infinite set of vector fields, ζ_n and $\bar{\zeta}_n$, can be defined. These **'large diffeomorphisms'** act non-trivially at the horizon and generate a $\mathrm{Vir}_{\mathrm{L}} \otimes \mathrm{Vir}_{\mathrm{R}}$ algebra, the symmetry algebra of a 2D CFT.

$$\zeta(\epsilon) = \epsilon\partial_+ + \frac{1}{2}\partial_+\epsilon\, y\partial_y$$

Quantifying Symmetries with Conserved Charges

The physical effect of these diffeomorphisms is captured by their associated conserved charges. These charges are calculated as surface integrals on the horizon bifurcation surface using the covariant phase space formalism.

$$\delta Q = \delta Q_{IW} + \delta Q_X$$

An Obstruction to a Consistent Algebra

The standard Iyer-Wald charge calculation reveals a critical issue. The central term in the Virasoro algebra, which measures the quantum anomaly, is found to depend on the black hole's temperatures. This violates a theorem requiring it to be constant.

$$KIW_{n,m} = 2J \frac{T_R}{T_L + T_R} m^3 \delta_{n+m}$$

Restoring Consistency with a Counterterm

Mathematical consistency of the charge algebra is restored by including a specific ‘Wald-Zoupas’ boundary counterterm, **X**, in the definition of the charge. This term is constructed to precisely cancel the unphysical temperature dependence.

$$X = 2dx^a h_a^b \Omega_b$$

The Final Central Charges

After applying the counterterm, the calculation yields constant, well-defined central charges for both the left- and right-moving sectors of the Virasoro algebra.

$$c_L = 12J \qquad c_R = 12J$$

Reproducing the Area Law from Symmetry

Assuming the black hole microstates form a 2D CFT, the Cardy formula can be used to count them. Combining the derived central charges (c_L, c_R) with the black hole temperatures (T_L, T_R) perfectly reproduces the Bekenstein-Hawking entropy.

$$S_{\text{Cardy}} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = 2\pi M_{r+} = S_{\text{BH}}$$

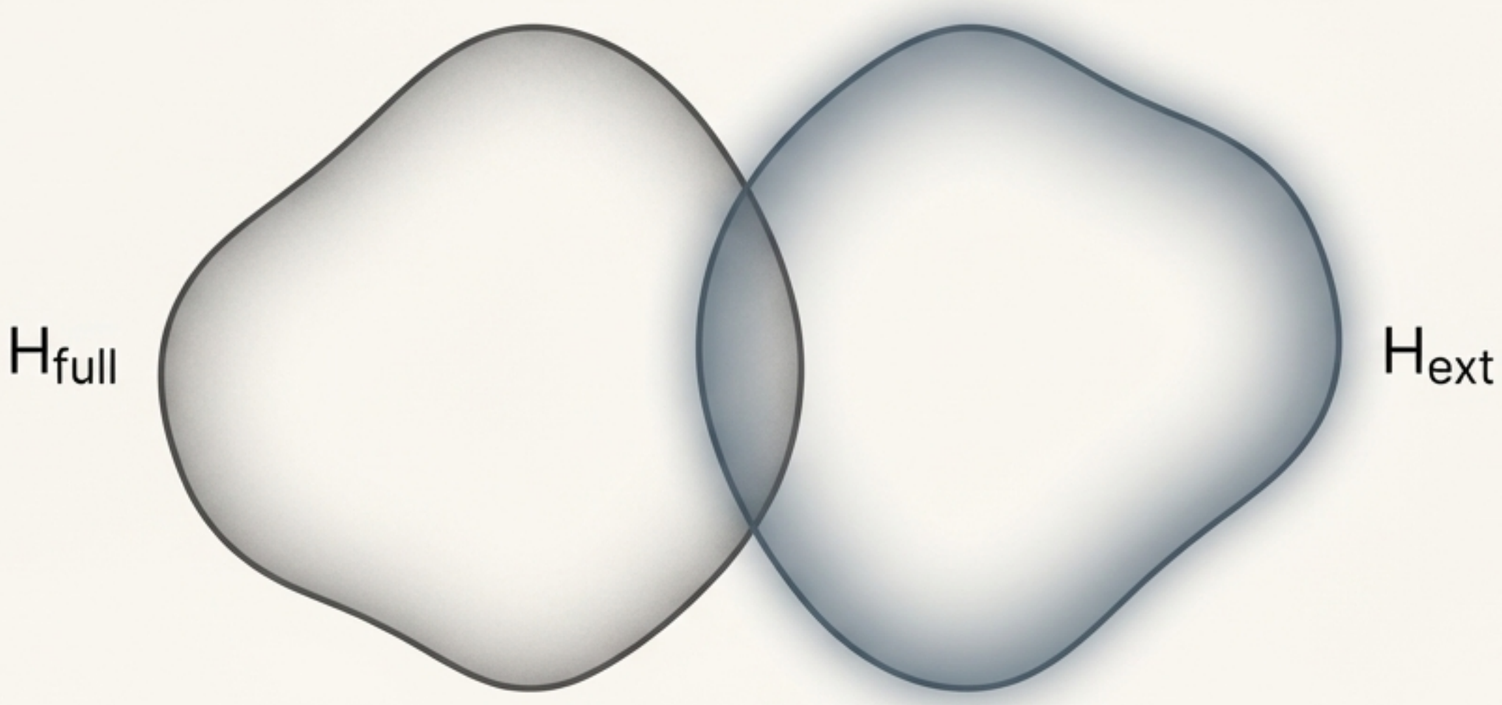
Boundaries of the Approach

This result provides strong evidence, but is not a final proof. It relies on the assumed applicability of the Cardy formula and a specific, non-unique construction of the counterterm.

"For these reasons our work might be regarded as incremental evidence for, but certainly not a demonstration of, the hypothesis that hidden conformal symmetry explains the leading black hole microstate degeneracy."

A Deeper Implication for the Information Paradox

This framework suggests that a black hole's microstates are entirely encoded by "soft hair" on the exterior. This implies the interior Hilbert space is not independent, challenging the standard view of spacetime factorization.



Summary of the Argument

The Problem of Microscopic Entropy → A Hidden Conformal Symmetry → Realized by Large Diffeomorphisms at the Horizon → Whose Charges Require a Counterterm for Consistency → Yielding Central Charges c = 12J → Which Reproduce the Bekenstein-Hawking Area Law via the Cardy Formula.

Source

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