Analysis: The Planck Constant (h)

Archival Documentation

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Abstract

This analysis examines the physical and ontological significance of the Planck constant. The investigation follows a strict 4-layer model: Incepta (History), Axiomata (Definition), Formalis (Application), and Cognitio (Interpretation).

I. INCEPTA: THE DIVERGENCE OF CLASSICAL PHYSICS

Towards the end of the 19th century, theoretical physics faced a fundamental anomaly known as the "Ultraviolet Catastrophe." The subject of investigation was the spectral energy distribution of black-body radiation in thermal equilibrium.

Classical electrodynamics and statistical mechanics, formalized in the Rayleigh-Jeans law, predicted that the spectral energy density ρ increases with the square of the frequency ν:

$$\rho(\nu, T) \propto \nu^2 k_B T \tag{1}$$

This led to the physically impossible consequence that an ideal black body would emit an infinite amount of energy at high frequencies. Experimental data significantly contradicted this prediction. This discrepancy marked the limit of classical continuum physics. Max Planck's introduction of a heuristic formula in 1900 required the assumption that energy exchange processes cannot occur continuously.

II. AXIOMATA: DEFINITION AND QUANTIZATION

The Planck constant h is defined as a fundamental physical constant that quantizes the action (energy $\times$ time) of physical systems.

Numerical Value

Since the redefinition of the SI unit system (2019), the value is fixed exactly:

$$h = 6.62607015 \times 10^{-34}\,\text{Js} \tag{2}$$

Postulate of Energy Quantization

Planck postulated that oscillators can only emit or absorb energy in discrete packets (quanta). The energy E of a quantum is proportional to the frequency ν:

$$E = h \cdot \nu = \hbar \cdot \omega \tag{3}$$

where $\hbar = \frac{h}{2\pi}$ represents the reduced Planck constant. This axiomatization discards the classical assumption of the infinite divisibility of energy.

III. FORMALIS: OPERATIVE APPLICATION AND STABILITY

In the formal structure of quantum mechanics, $\hbar$ functions as a scaling factor for commutator relations and ensures the stability of matter.

Heisenberg Uncertainty Principle

Certain pairs of observables (e.g., position x and momentum p) cannot be simultaneously determined with exact precision:

$$\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2} \tag{4}$$

If $h = 0$, classical mechanics would be universally valid.

Derivation of Atomic Stability (Bohr Model)

Classically, electrons would spiral into the nucleus due to radiation losses. h prevents this through the quantization of angular momentum L.

1. **Quantization Condition:**

$$L = m_e v r = n\hbar \implies v = \frac{n\hbar}{m_e r} \quad (n \in \mathbb{N}) \tag{5}$$

2. **Force Equilibrium (Coulomb = Centripetal):**

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{m_e v^2}{r} \tag{6}$$

3. **Solving for Radius r:** Substituting v yields the orbital radius r_n:

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} \cdot n^2 \tag{7}$$

Conclusion: Since $n \geq 1$, the radius r is bounded from below ($r_1 \approx 0.53$ Å). A collapse ($r = 0$) is physically forbidden by the existence of h.

IV. COGNITIO: ONTOLOGICAL IMPLICATIONS

The existence of h forces the transition from an analog to a discrete universe.

Discreteness of Spacetime

The principle *Natura non facit saltus* is invalid at the Planck scale. State changes occur abruptly. This suggests a granular structure of spacetime.

Information Theory

Since action is discrete, the information content of a volume of space is finite (Bekenstein bound). The universe does not operate on infinitely divisible continua, but on discrete units of information (qubits). Reality is fundamentally computationally structured.